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SMG-8046 RF-electronics preparatory I

Small Exam 1, 19.09.10 at 9.15

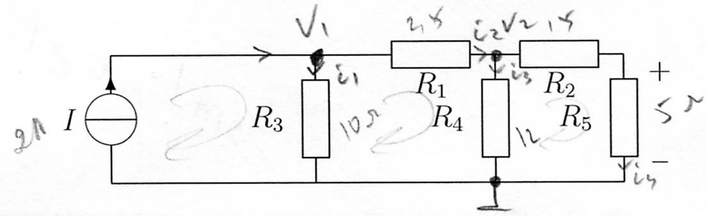
Answer to three of the four questions. Each question gives maximum of 5 points.

1. (a) i. Assume that the voltage over R_5 is 1V. What is value of I then?

   ii. Assume that $I = 2$A. Find voltage over R_5. Determine also the voltages and currents of all the other resistors.

   Let $R_1 = 2.5\Omega$, $R_2 = 15\Omega$, $R_3 = 10\Omega$, $R_4 = 12\Omega$, and $R_5 = 5\Omega$.

   $V_2 = V_{R_2} + V_{R_5}$



   (b) Are following true or not. Include proper reasoning to your answer.

   i. Any five vectors in $\mathbb{R}^5$ form a basis for $\mathbb{R}^5$.

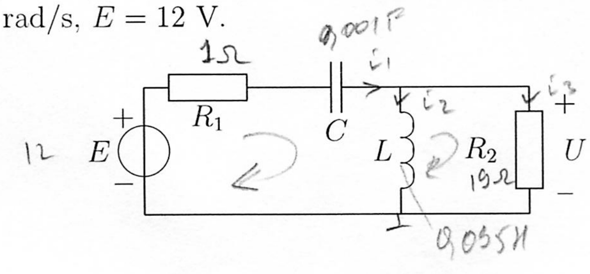
   ii. A basis in a vector space is unique.

   iii. If A is a 3×3 matrix and $\det(A)$ $=0$, then the rows of A are linearly dependent vectors of $\mathbb{R}^3$.

2. (a) Let us assume that space V has basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$. Let us also assume that for two linear transformations $T_1 : V \to W$ and $T_2 : V \to W$ holds that $T_1\mathbf{v}_i = T_2\mathbf{v}_i = \mathbf{w}_i$ for all $i = 1, 2, \ldots, n$. Show that then for any vector $\mathbf{v} \in V$ holds that $T_1\mathbf{v} = T_2\mathbf{v}$.

   (b) i. Find the voltage U. Let $R_1 = 1\Omega$, $R_2 = 19\Omega$, $L = 0,095$H, $C = 0,001$F, $\omega = 100$ rad/s, $E = 12$ V.

   $U = 19\angle 90°$



   ii. Comment on your result, does it seem to be a bit unexpected?

3. (a) Determine rank and nullity of matrix A.

$$A = \begin{bmatrix} 3 & 0 & 2 \\ -6 & 42 & 23 \\ 21 & -21 & 0 \end{bmatrix}$$

   → rank ? = 2

   nullity = 1 ?

   rank + nullity = n   3

   0

   (b) Let $A \in \mathbb{R}^{m \times n}$ and let us define a transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ as

$$T_A(\mathbf{x}) = A\mathbf{x}$$

   where $\mathbf{x} \in \mathbb{R}^n$. Is T_A linear or not? Give a proper verification.

TURN OVER

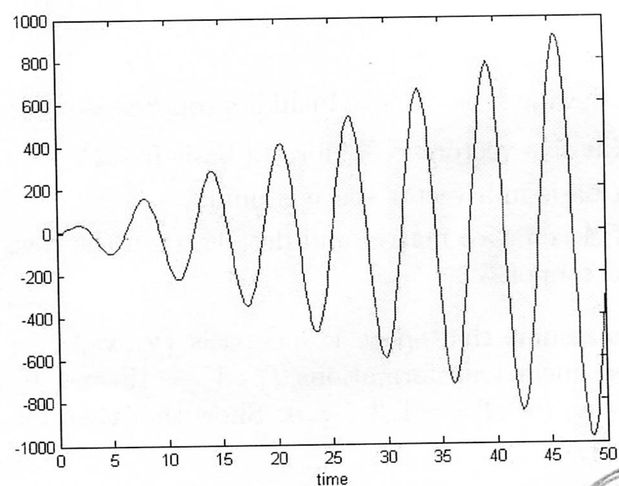
4. (a) Solve the initial value problem

$$y'' + 3y' - 10y = 0, \quad y(0) = 3, \quad y'(0) = -1.$$

Show also that you have obtained linearly independent solution(s).

(b) Look at the picture below which is related to solution of a 2nd order differential equation. Based on the picture, estimate what happens when time increases further. How could you describe the system in question? Could you deduce and write down a possible equation that would characterize the solution?

-sin! not e!



$d f \sin(2\pi f t)$!