Please answer **four** out of the following five questions, on a separate paper. Note that there are two pages in the exam. No books. *non-programmable calculators are allowed.*

A Collection of Formulas will be handed. Kaavakokoelma jaetaan.

1. Consider the optimization problem:

$$\min f(x,y) = (x-2)^2 + y^2$$

s.t.

$$x^2 + y^3 \leq 1$$

   (a) Is the feasible set convex?

   (b) Is the cost-function convex?

   (c) Calculate the optimum and demonstrate that it is a KKT-point.

2. Assume that we have the following linear problem:

$$\min c^T x$$

s.t.

$$Ax = b$$

$$x \geq 0$$

   (a) Suppose that we wish to use the Simplex algorithm. To do so, we need one basic feasible solution to the LP. Explain how to find one.

   (b) Consider the reduced cost $\hat{c}_N$ (formula given in the formula sheet). Show (using algebra), that its component i is equal to the change in the cost function if we increase the value of the null-variable x_i by one. I.e., assume you have a basic feasible solution x_b and you move from this point, all the while maintaining the constraint $Ax = b$.

3. Consider the unconstrained problem

$$\min f(x,y) = (2-x)^2 + 6(y-x^2)^2$$

   (a) Find the minimum of this function analytically. (Hint: It is not hard, if you really look at when it can be minimum. Or solve for the zero of the gradient, but that is going to be hard.

   (b) Consider finding the minimum using one of the methods. Start from any point (other than the optimum) and calculate one iteration. If you use the gradient method, be sure to formulate the minimization problem solving the step length.

   (c) Demonstrate that the function is not convex. (By any means you like.)

4. Let $f, g : X \to \mathbb{R}$ be two convex functions. Which of the following claims hold? Prove or disprove (i.e., give an example where the claim is false)

   (a) The function $af + bg$ is convex, when $a, b \in \mathbb{R}$.

   (b) The set $\{x | f(x) < g(x)\}$ is convex.

   (c) The function $h(x) = \max\{f(x), g(x)\}$ is convex.

5. Suppose we have a constraint problem

$$\min x^2 + y$$

s.t.

$$x^2 - y^2 = 1$$

   (a) Formulate the KKT-conditions for this problem.

   (b) solve the problem either by solving the KKT-equation or by using Lagrange multipliers. (Hint: it's not too hard, consider the factors)

   (c) What is the minimum of the function?