MAT-62756 Graph Theory Examination 19.12.2013

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NB This is a closed-book exam, no material is allowed. Nonprogrammable calculators are allowed.

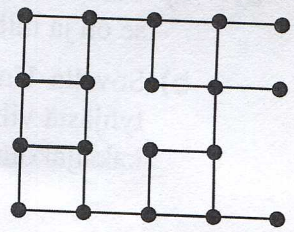
1. **a)** Show that for each number $n > 1$ there is a connected graph with n vertices all of them having different degrees.

b) Is this possible for any $n > 1$ if it is required that the graph is simple but not necessarily connected? Explain your reasoning!

2. Explain briefly what is **a)** the adjacency matrix, **b)** the incidence matrix, **c)** the cut matrix, **d)** the circuit matrix, **e)** the fundamental cut matrix and **f)** the fundamental circuit matrix of a digraph.

3. **a)** The Hungarian Algorithm for bipartite graphs, how it works and what is its function.

b) Apply the Hungarian Algorithm to the graph on the right starting from the empty edge set. Explain carefully each step you take! (The graph is bipartite.)



4. **a)** Give an example of a case where a transport network has a cycle and the Ford–Fulkerson Algorithm gives a positive flow for each arc of the cycle. Explain the steps of the algorithm in detail.

b) Show that if a transport network has cycles then there is a maximum flow such that in each cycle there is at least one arc with a zero flow.

(Flow around a cycle may in fact be useful in some situations (e.g. as a storage), but usually it just eats resources and should be removed. Algorithmically this is a bit tricky.)

5. The Davidson–Harel Algorithm, how it works and what is its function.

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