ELT-47426 Transmission lines and waveguides

Small Exam II, Feb 28th 2014. Answer to all questions.
Jari Kangas

1. (a) A rectangular waveguide with dimensions $a = 1.5$ cm, $b = 0.8$ cm is filled with a dielectric with permeability μ_0 and relative permittivity 4. The magnetic field inside the guide is given as

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin(\pi 10^{11}t - \beta z)\ \text{A/m}.$$

   Find: (4 p. in total)

   i. The mode that is propagating inside the guide.
   ii. The cutoff frequency. $f_c = 2.88 \cdot 10^{10}$ Hz
   iii. The propagation constant. $\beta = 1722.02$
   iv. The wave impedance. TE = 229 Ω   TM = 158 Ω

   (b) Consider rectangular air–filled waveguide. Determine criteria for having maximum bandwidth for such a structure such that it also offers maximum power withstand for dominant mode. Determine the corresponding cut–off wavenumber. (4 p.)

2. Look at the pictures of a simulation case on the other paper. Explain what kind of settings (parameters, conditions) are inherent in the simulation case. Consider them from EM field analysis point of view. (4 p.)

3. (a) Consider transmission line and waveguide analysis, your task is to explain key differences between them. (3 p.)

   (b) Explain how to check orthogonality of different modes in a spherical cavity. Reason also why orthogonality of different modes is of interest and where it can be used. (3 p.)

Miscellaneous information

- Resonance frequency of a rectangular cavity resonator can be expressed as

$$f_{cp} = \frac{1}{2\pi\sqrt{\epsilon\mu}}\sqrt{k_c^2 + \left(\frac{p\pi}{d}\right)^2}$$

where $\mu_0 = 4\pi * 10^{-7}$H/m and $\epsilon_0 = 8.854 * 10^{-12}$F/m.

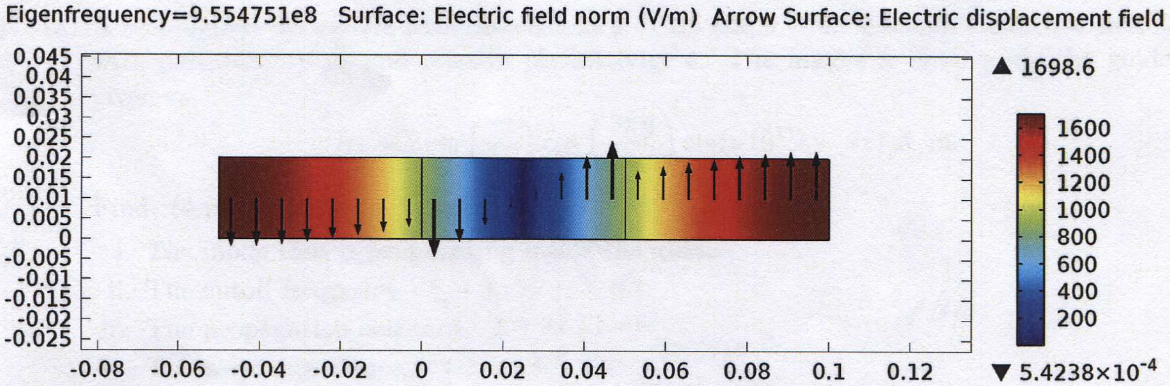
For rectangular waveguides average power is given and field components are related as:

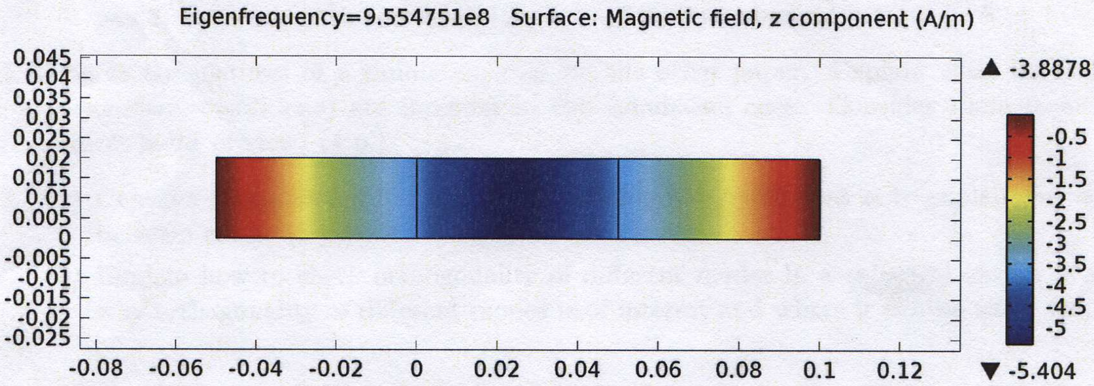
$$\langle P_{mn}\rangle = \frac{\beta_{mn}^2 a^3 b^3}{8\pi^2\eta_{mn}^{\text{TM}}(n^2a^2 + m^2b^2)}E_{zm}^2 \quad \text{and} \quad \langle P_{mn}\rangle = \eta_{mn}^{\text{TE}}\frac{\beta_{mn}^2 a^3 b^3 H_{zm}^2}{8\pi^2(n^2a^2 + m^2b^2)}$$

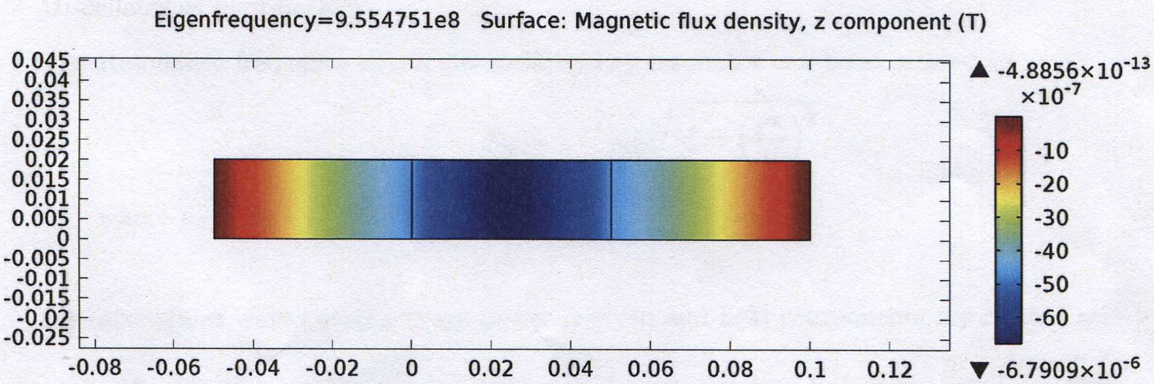
$$\mathbf{H}_t = \frac{1}{\beta^2 - k^2}(j\beta\nabla_t H_z + j\omega\epsilon\,\hat{\mathbf{z}}\times\nabla_t E_z), \quad \mathbf{E}_t = \frac{1}{\beta^2 - k^2}(j\beta\nabla_t E_z - j\omega\mu\,\hat{\mathbf{z}}\times\nabla_t H_z).$$

Constants in free space and some formulas:

- speed of light in vacuum $c = 2.99792458 * 10^8$m/s $\approx 3 * 10^8$m/s
- intrinsic impedance $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi\ \Omega$
- $e^x \approx 1 + x$ if $|x|$ small


Eigenfrequency=9.554751e8 Surface: Electric field norm (V/m) Arrow Surface: Electric displacement field
▲ 1698.6
▼ 5.4238×10^-4


Eigenfrequency=9.554751e8 Surface: Magnetic field, z component (A/m)
▲ -3.8878×10^-7
▼ -5.404


Eigenfrequency=9.554751e8 Surface: Magnetic flux density, z component (T)
▲ -4.8856×10^-13
×10^-7
▼ -6.7909×10^-6