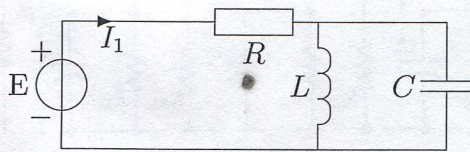


Answer to all three questions.

Jari Kangas

1. (a) Explain how  $\mathbf{E}$ -field is defined and which factors affect on its value. Could there be cases when  $\mathbf{E}$ -field is equal to zero in a (sub)domain? If so, describe such cases briefly (aim to highlight what is special about them).
- (b) Let us consider two charges,  $Q_1$  at  $(1, -1, -3)$  [m] and  $Q_2$  at  $(3, -3, 2)$  [m].  $Q_1$  experiences force  $\mathbf{F} = 8\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$  [N] and charge of  $Q_1$  is known to be  $300\mu\text{C}$ . Determine charge of  $Q_2$ . To simplify your calculations assume that  $\epsilon_0 \approx \frac{10^{-9}}{36\pi} \text{F/m}$
2. (a) In the circuit below assume that  $I_1 = (2 - j4)\text{A}$ . Find the real and reactive powers given by the source. Assume that  $R = 1\Omega$ ,  $C = 5\mu\text{F}$ ,  $L = 1\text{mH}$ , and  $\omega = 1000 \text{ rad/s}$ .



- (b) Consider a graph whose loop matrix is

$$B = \begin{bmatrix} -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

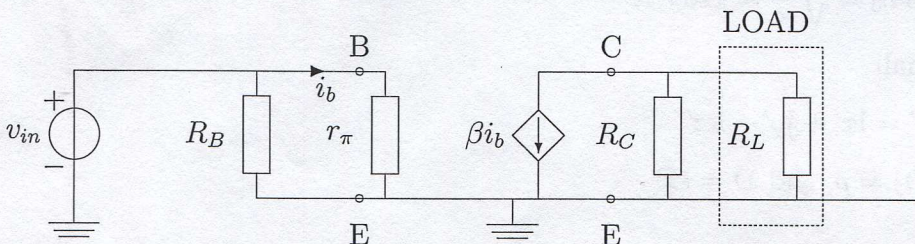
Draw a corresponding graph, if edges IV and V belong to cotree, and the loops are clockwise.

- (c) Consider following statement:

“In case of capacitors, say parallel plate capacitors, no charge is carried through the insulating material between the plates by direct current (DC), but alternating current (AC) can carry charge through the material.”

Do you agree or disagree with this statement. *To get points, support your answer by an argument or an example.*

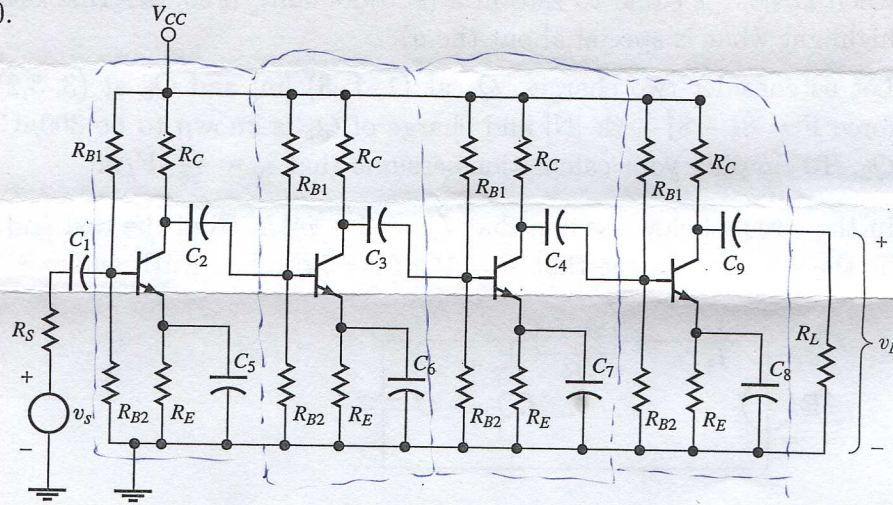
3. (a) Below is an equivalent circuit for a common emitter amplifier. The amplifier is used in a multistage amplifier as shown in picture (a) in the figure on the opposite side. In the equivalent circuit  $R_B = R_1 || R_2$ , output is taken over  $R_L$ .



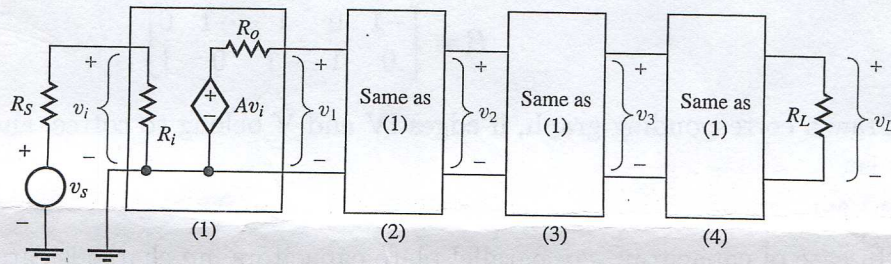
- i. Determine open-circuit voltage amplification  $A$ .
- ii. Determine the input resistance  $R_i = v_{in}/i_{in}$ , approximate its value by assuming that  $R_B \gg r_\pi$  (use this approximation the question 3. (b) i., see opposite side).
- iii. Determine the output resistance  $R_o$  which is defined as

$$R_o = - \frac{v_{out}^{\text{when output terminals open}}}{i_{out}^{\text{when output terminals shorted}}}$$

- (b) i. The amplifier is used in a multistage amplifier as shown below. The multistage amplifier has four stages, each of them are similar. Determine the overall gain using simplified amplifier model as shown in picture (b) below. Use the equivalent circuit shown in the picture (b) and the expressions that you derived in item 3.(a).
- ii. Derive numerical estimate for the overall gain in case  $R_C = R_L = r_\pi = R_S$  and  $\beta = 100$ .



(a)



(b)

Constants in free space and some formulas:

- dielectric constant  $\epsilon_0 \approx 8.854 \times 10^{-12} \text{F/m}$
- permeability  $\mu_0 \approx 4\pi \times 10^{-7} \text{H/m}$
- speed of light  $c \approx 2.997925 \times 10^8 \text{m/s}$
- intrinsic impedance  $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi \Omega$
- $e^x \approx 1 + x$  if  $|x|$  small
- $\mathbf{r} = ix + jy + kz, \mathbf{r}' = ix' + jy' + kz'$
- $\text{curl}(\mathbf{E}) = 0, \text{div}(\mathbf{D}) = \rho, \text{ and } \mathbf{D} = \epsilon\mathbf{E}$
- $\varphi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\mathbf{r}-\mathbf{r}'|}$
- $P_{ave} = \frac{1}{2} U_{max} I_{max}^* = \frac{U_{max}}{\sqrt{2}} \frac{I_{max}^*}{\sqrt{2}} = U_{rms} I_{rms}^*$
- $\int_S \text{curl}(\mathbf{F}) \cdot \mathbf{n} da = \int_{\partial S} \mathbf{F} \cdot d\mathbf{l}$
- $\int_V \text{div}(\mathbf{F}) dv = \int_{\partial V} \mathbf{F} \cdot \mathbf{n} da$
- $\int_C \text{grad}(f) \cdot d\mathbf{l} = f(b) - f(a)$
- $\text{grad}(f) = \mathbf{i} \frac{\partial f}{\partial x} + \mathbf{j} \frac{\partial f}{\partial y} + \mathbf{k} \frac{\partial f}{\partial z}$