

Use of own programmable calculator is allowed.

**Problem 1** (max 8 points)

Power stage of a voltage-fed inverter with output-current control is as shown in Figure 1.

- Develop the inverter average model in dq-domain at open-loop,  $\left( \langle i_{in} \rangle = \frac{3}{2} d_d \langle i_{Ld} \rangle + \frac{3}{2} d_q \langle i_{Lq} \rangle \right)$ .
- Draw the equivalent linear circuit in dq-domain (DC input port and two AC output ports).
- How do you have to modify the duty ratio  $d$  and  $q$ -components (how to define  $x_d$  and  $x_q$ ) to make the inverter output currents independent of the grid voltage  $d$  and  $q$ -components? Justify the result using the average model.
- How do you realize decoupling of the current  $d$  and  $q$ -components? Use the average model to obtain necessary control laws / scaling coefficients. Correct answers earn points only if you can justify them using the average model.

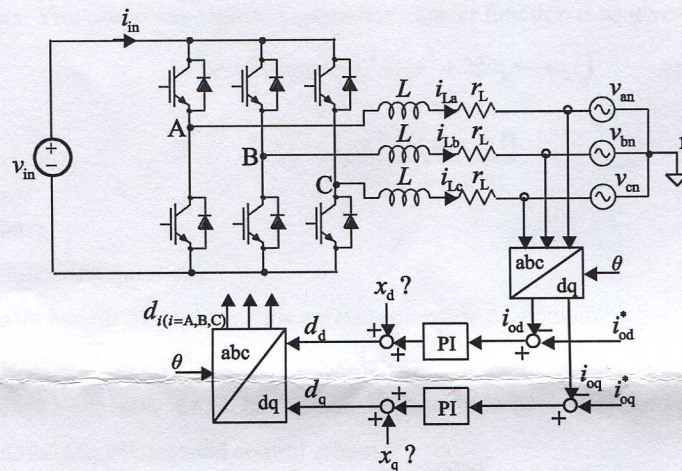


Figure 1: Voltage-fed inverter with output-current control.

**Problem 2** (max 4 points)

Instantaneous apparent power can be defined as the product of voltage space-vector and the complex-conjugate of current space-vector in the stationary reference frame as in (1).

- Define real and imaginary power in the synchronous reference frame, i.e., in the dq-domain. The space vector is assumed to rotate at the fundamental grid frequency  $\omega_s t$ . (Useful formulas:  $x^{a\beta} = x^{dq} \cdot e^{j\omega_s t}$ ,  $(x \cdot y)^* = x^* \cdot y^*$ ,  $(x^{j\theta})^* = x^{-j\theta}$ ,  $j^2 = -1$ )
- Explain based on the previous result how the real and imaginary power produced by three-phase inverter can be controlled independently.

$$s = v^{a\beta} \cdot (i^{a\beta})^* \quad (1)$$

**Problem 3** (max 6 points)

The control block diagram of the phase-locked-loop is as shown in Figure 3. The feedforward term  $\omega_{ff}$  is a constant which improves the start-up. The Park's transformation can be linearized as  $\hat{v}'_q = \hat{v}_q - V_d \hat{\theta}$  where  $\hat{v}_q$  denotes the ideal grid voltage q-component.

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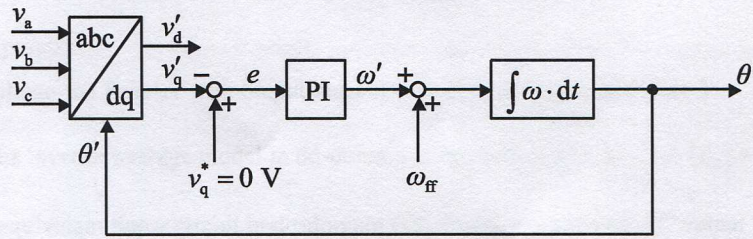


Figure 3: Phase-locked-loop.

- Draw the linearized control block diagram and give the control loop gain of the PLL.
- Solve transfer function from the reference input to the controlled variable  $\hat{v}'_q$  in dq-domain.
- The transfer function from reference to the controlled variable can be written as a second-order system as in (2). Find out the damping ratio  $\zeta$  and natural frequency  $\omega_n$  in terms of controller parameters. You can assume that the controller transfer function is as given in (3).

$$G = (2\zeta\omega_n s + \omega_n^2) / (s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{2}$$

$$G_c = \frac{(-1) \cdot K (s/\omega_z + 1)}{s} \tag{3}$$

**Problem 4** (max 6 points)

Give short answers to following questions.

- What is the main benefit of implementing current control in dq-domain?
- Why do you need to solve steady-state operating point?
- Can you stabilize a converter which has an unstable pole in its control dynamics? How?
- Give a short definition of cascaded control scheme?
- How does phase-locked-loop affect the output impedance of a three-phase inverter?
- How does grid-voltage feedforward affect the output impedance of a three-phase inverter?

**Problem 5** (max 6 points)

Three-phase LCL-filter is shown in Figure 4. Solve the average state-space model of the filter in the dq-domain. You can assume that the three-phase input and output voltages are balanced. Draw the electrical circuits of the filter in dq-domain (separately for d and q-components),  $(\mathbf{x}^{\alpha\beta} = \mathbf{x}^{dq} \cdot e^{j\omega_s t})$ ,  $(\mathbf{T}^{abc \rightarrow \alpha\beta} \cdot [k \ k \ k]^T = [0 \ 0 \ k]^T)$ .

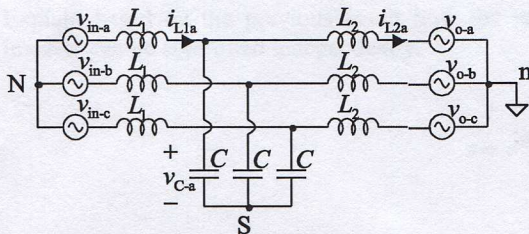


Figure 4: Three-phase LCL-type filter.

Hints:

$$\frac{d}{dt} i_{L1}^{dq} = f(v_{NS}, v_{in-d}, v_{in-q}, v_{C-d}, v_{C-q})?$$

$$\frac{d}{dt} v_C^{dq} = f(i_{L1d}, i_{L1q}, i_{L2d}, i_{L2q})?$$

$$\frac{d}{dt} i_{L2}^{dq} = f(v_{nS}, v_{C-d}, v_{C-q}, v_{o-d}, v_{o-q})?$$