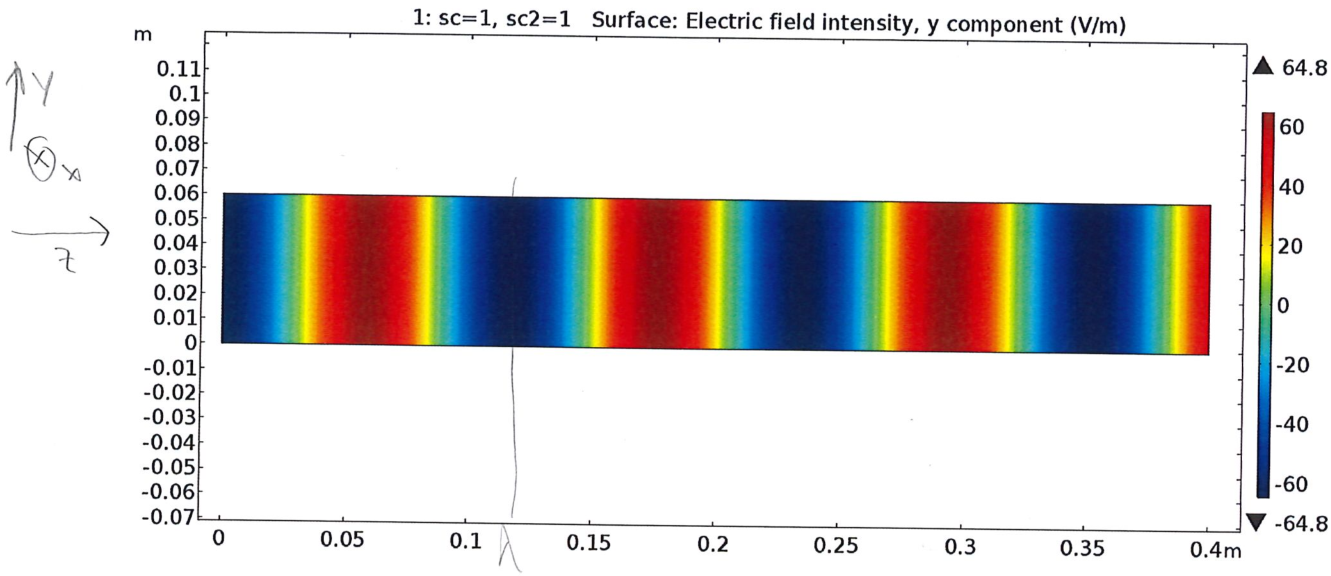


Figure 1: Parallel plate transmission line, E_y -component at $f = 850$ MHz.

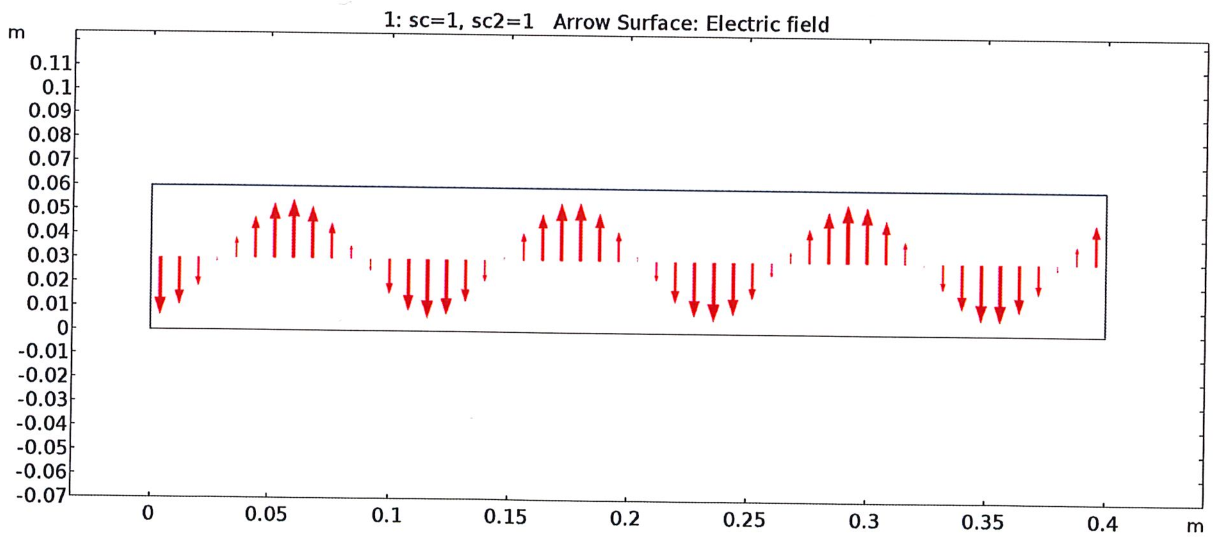


Figure 2: Parallel plate transmission line, $\mathbf{E}$ –field at $f = 850$ MHz.

$k = \omega\sqrt{\epsilon\mu}$ $\quad f_c = \frac{1}{2\pi\sqrt{\epsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{L}\right)^2}$ $\quad V_g = \frac{k}{c\beta}$

$f = \frac{k}{2\pi\sqrt{\epsilon\mu}}$

COMM.RF.430 Transmission lines and waveguides

Small Exam II, February 24th 2023. Answer to all questions.
Jari Kangas

1. (a) A www-page states that

   "Transmission lines can be considered as special types of waveguides".

   Evaluate the definition i.e. comment on the definition based on what you know about waveguides and transmission lines.
   (3 p.)

   (b) Correct or incorrect? *To get points, support your answer by an argument or an example.*

   i. First a fact: For TM–modes in a cylindrical waveguide one finds solution

   $$E_z(r, \phi) = CJ_n(k_c r)\cos n\phi$$

   in the cylindrical r, ϕ, z –coordinate system.
   Then the statement: The cutoff frequencies can be calculated if zeros of the Bessel function are known. (1 p.)

   ii. Any wave in a coaxial cavity resonator can be expressed as linear combination of the resonance modes. (1 p.)

2. This question is related to Figures 1,2, and 3. Geometry of the structure is the same in all cases. In one of the pictures the structure is filled with air, in the two others with a dielectric (see the captions for further info).

   (a) Explain the mode shown in Fig. 1. Reason your answer carefully.
   (b) Use the Figures to find the relative permittivity of the dielectric inside the cavity.
   (c) Use the Figures to find the second longest dimension of the cavity, if the longest dimension is 0.18 m.

   In total 4 p.

3. A collegue of yours asks for advice on rectangular waveguides. The collegue needs to feed an antenna with the dominant mode. The antenna is supposed to operate at 7.5 GHz (±300 MHz).

   (a) From the list provided find a waveguide suitable for this task (the waveguides are air–filled). At the bottom of the datasheet you find a picture that shows the geometry parameters.
   (b) Find the propagation constant and the wavelength of the lowest mode at 4.0 GHz, 7.9 GHz, and 16.0 GHz. What could you conclude from the results?
   (c) Comment on the usable bandwidth of the chosen waveguide.
   (d) Outline briefly the analytical process to determine modes in the rectangular waveguide. HINT: This is rather general question, so aim to focus on the main steps.

   In total 5 p.

$\beta = \sqrt{k^2 - k_c^2}$

Constants in free space and some formulas:

- dielectric constant $\epsilon_0 \approx 8.854 * 10^{-12}$ F/m
- permeability $\mu_0 = 4\pi * 10^{-7}$ H/m
- speed of light $c \approx 2.997925 * 10^8$ m/s
- intrinsic impedance $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi\ \Omega$
- $e^x \approx 1 + x$ if $|x|$ small

Miscellaneous information about rectangular waveguides etc.

- $k = \omega\sqrt{\mu\epsilon}$, $\beta^2 = k^2 - k_c^2$, $\lambda = \frac{2\pi}{\beta}$,
- $-k_x^2 - k_y^2 + k_c^2 = 0$, where $k_x^2 = \left(\frac{m\pi}{a}\right)^2$, $k_y^2 = \left(\frac{n\pi}{b}\right)^2$
- For TE–modes fields are related as (where F_z is suitable scalar function, for example a trigonometric function):

$$E_x = -\frac{1}{\epsilon}\frac{\partial F_z}{\partial y}, \qquad H_x = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 F_z}{\partial x \partial z}, \qquad E_y = \frac{1}{\epsilon}\frac{\partial F_z}{\partial x}, \qquad H_y = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 F_z}{\partial y \partial z}.$$

- For good conductors skin depth is

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

- For TE_{101}–mode:

$$Q_{101} = \frac{\pi f_{101}\mu_0 abd(a^2 + d^2)}{R_s\left[2b(a^3 + d^3) + ad(a^2 + d^2)\right]} \qquad \text{where} \qquad R_s = \frac{1}{\delta\sigma}.$$

-

$$\mathbf{H}_t = \frac{-1}{k_c^2}(\mathrm{j}\beta\nabla_t H_z + \mathrm{j}\omega\epsilon\,\hat{\mathbf{z}} \times \nabla_t E_z),$$
$$\mathbf{E}_t = \frac{-1}{k_c^2}(\mathrm{j}\beta\nabla_t E_z - \mathrm{j}\omega\mu\,\hat{\mathbf{z}} \times \nabla_t H_z),$$

where $\nabla_t = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}$ and

$$E_z(x,y) = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y),$$
$$H_z(x,y) = H_0 \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y).$$

- Conductivities:

Silver	$6.17 * 10^7$ [S/m]	Copper	$5.80 * 10^7$ [S/m]
Brass	$1.57 * 10^7$ [S/m]	Iron	10^7 [S/m]

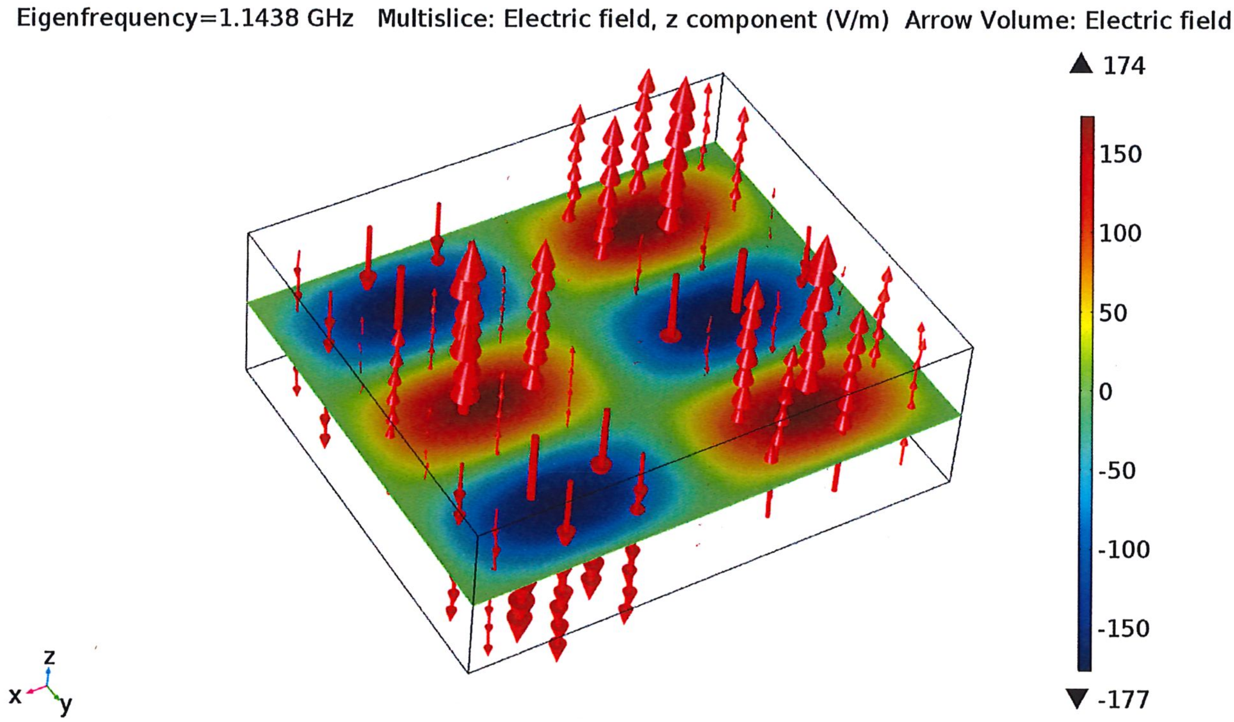


Figure 1: Pattern of a mode in a rectangular cavity resonator filled with a dielectric.

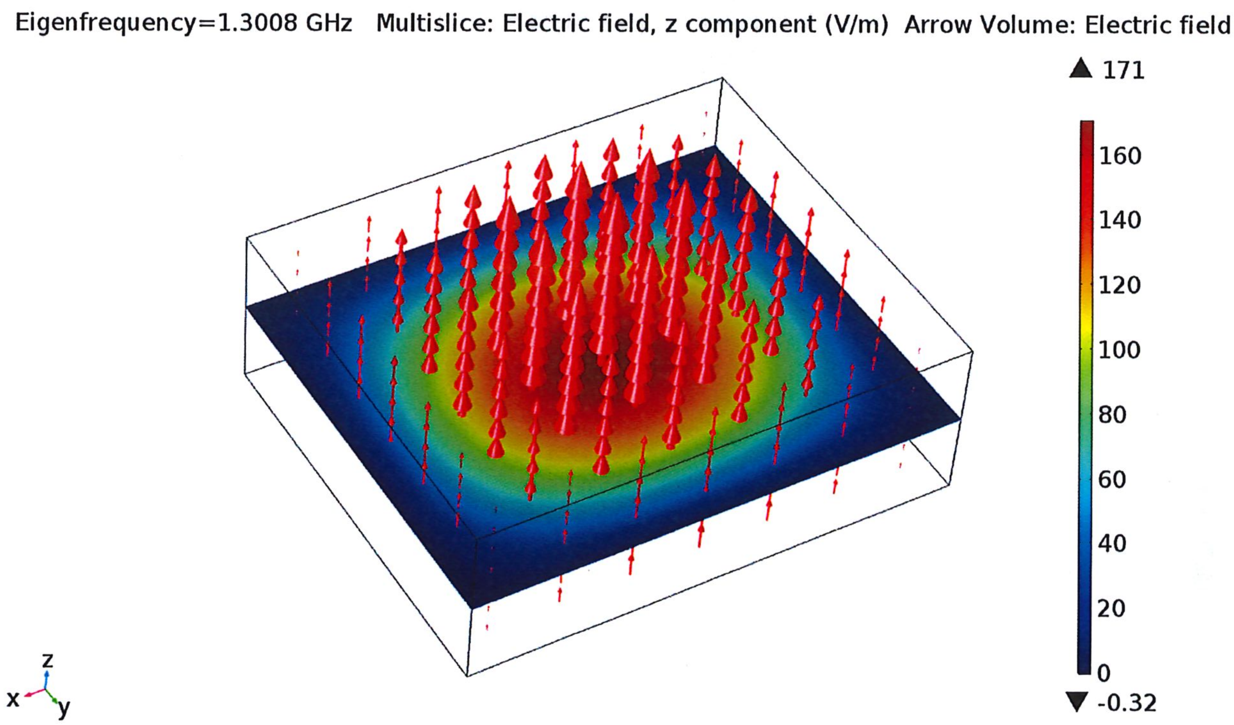


Figure 2: Pattern of a mode in a rectangular cavity resonator filled with air.

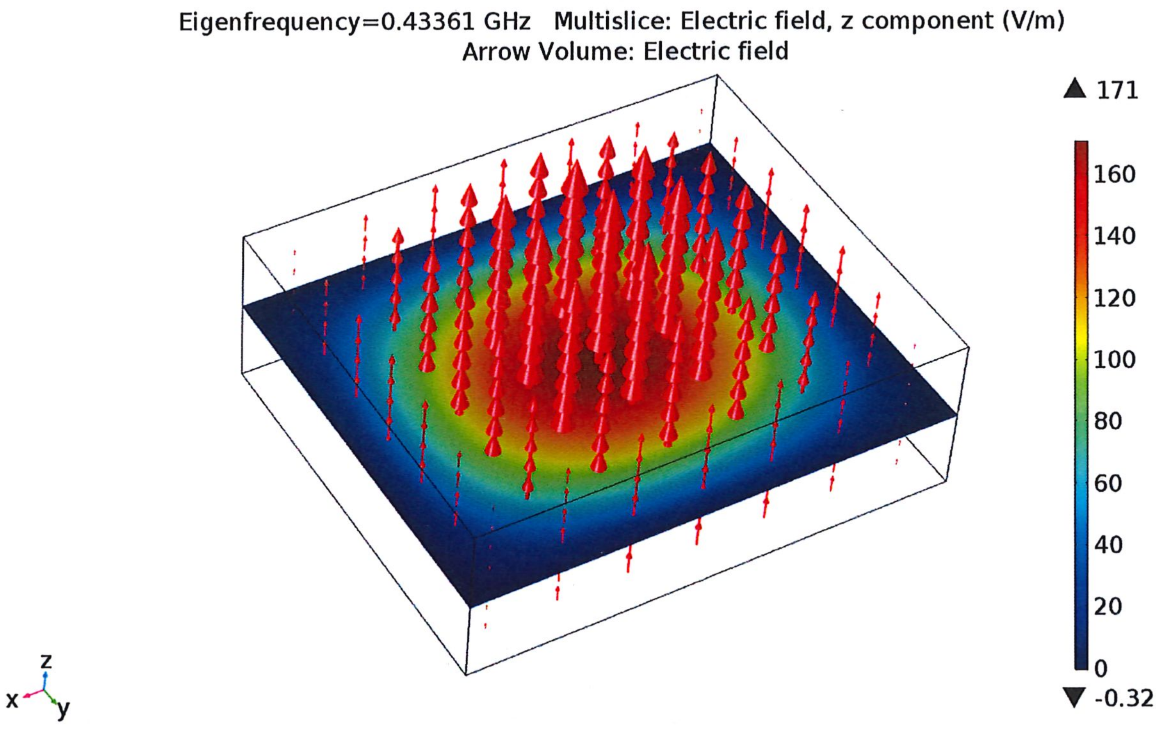


Figure 3: Pattern of a mode in a rectangular cavity resonator filled with a dielectric.

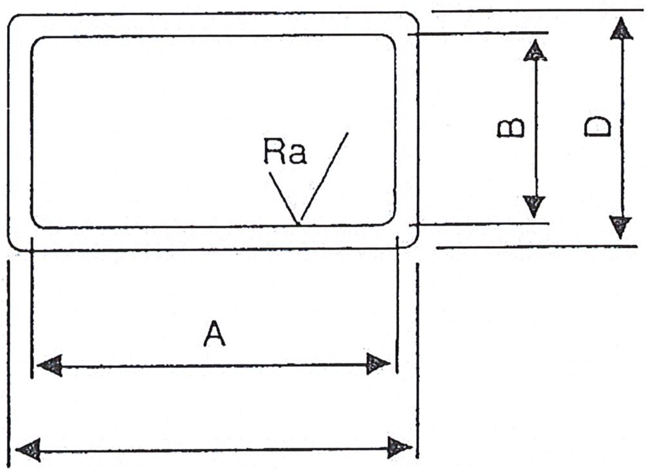
COMM.RF.430 Transmission lines and waveguides

FOR QUESTION NUMBER 3

STANDARD RECTANGULAR WAVEGUIDE

DATA SHEET No. T110

Waveguide Size Europe And USA			Internal Dimensions Ax B			
R	Wg	Wr	mm	tol.t	inches	tol.±
14	6	650	165.10x82.55	0.20	6.500x3.250	.008
22	8	430	109.22x54.61	0.14	4.300x2.150	.006
26	9A	340	86.36x43.18	0.11	3.400x1.700	.005
32	10	284	72.14x34.04	0.08	2.840x1.340	.004
40	11A	229	58.17x29.083	0.06	2.290x1.145	.003
48	12	187	47.55x22.149	0.05	1.872x0.872	.003
58	13	159	40.39x20.139	0.05	1.590x0.795	.002
70	14	137	34.85x15.799	0.04	1.372x0.622	.002
84	15	112	28.499x12.624	0.03	1.122x0.497	.002
100	16	90	22.86 x10.16	0.03	0.900x0.400	.001
120	17	75	19.05x9.525	0.02	0.750x0.375	.001
140	18	62	15.799x7.899	0.02	0.622x0.311	.0008
180	19	51	12.954x6.477	0.02	0.510x0.255	.0008
220	20	42	10.668x4.318	0.02	0.420x0.170	.0008
280	21	34	8.636x4.318	0.02	0.340x0.170	.0008
320	22	28	7.112x3.556	0.02	0.280x0.140	.0008
400	23	22	5.690x2.845	0.02	0.224x0.112	.0008
500	24	19	4.775x2.388	0.02	0.188x0.094	.0008
620	25	15	3.759x1.880	0.02	0.148x0.074	.0008
740	26	12	3.099x1.549	0.02	0.122x0.061	.0008
900	27	10	2.540x1.270	0.02	0.100x0.050	.0008



Length: 3050 mm (Other upon request)
Alloy: 6063 (Other upon request)

Straightness and twist:
DIN 17615

COMM.RF.300 Analysis of Electromagnetic Systems

$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$
$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$
$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

$\nabla f = \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} + \mathbf{k}\frac{\partial f}{\partial z}$
$\nabla(\psi + \phi) = \nabla\psi + \nabla\phi$
$\nabla(\psi\,\phi) = \phi\,\nabla\psi + \psi\,\nabla\phi$
$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$
$\nabla \cdot (\psi\mathbf{A}) = \psi\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla\psi$
$\nabla \times (\psi\mathbf{A}) = \psi\nabla \times \mathbf{A} + \nabla\psi \times \mathbf{A}$
$\nabla \cdot (\nabla \times \mathbf{A}) = 0$
$\nabla \times (\nabla\psi) = \mathbf{0}$
$\nabla \cdot (\nabla\psi) = \nabla^2\psi$
$\nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}) = \nabla^2\mathbf{A}$
$\nabla \cdot (\phi\nabla\psi) = \phi\nabla^2\psi + \nabla\phi \cdot \nabla\psi$

$\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$
$\mathbf{r}' = \mathbf{i}x' + \mathbf{j}y' + \mathbf{k}z'$

$\int_S \nabla \times \mathbf{F} \cdot \mathbf{n}da = \int_{\partial S} \mathbf{F} \cdot \mathbf{t}dl$
$\int_V \nabla \cdot \mathbf{F}\,dv = \int_{\partial V} \mathbf{F} \cdot \mathbf{n}da$
$\int_C \nabla f \cdot \mathbf{dl} = f(b) - f(a),\ \partial C = b - a$
$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\nabla \cdot \mathbf{D} = \rho$
$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$\varphi(\mathbf{r}) = \int_{V'} \frac{\rho(\mathbf{r}')}{4\pi\epsilon|\mathbf{r} - \mathbf{r}'|}dv'$
$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}') \times \mathbf{R}}{|\mathbf{R}|^3}dv'$

$e = -1.602 * 10^{-19}$ C $= -1.602 * 10^{-19}$ A·s
$m_e = 9.109 * 10^{-31}$ kg
$\epsilon_0 = 8.854 * 10^{-12}$ F/m $= 8.854 * 10^{-12}$ kg^{-1}·m^{-3}· s^4· A^2
$\mu_0 = 4\pi * 10^{-7}$ H/m $= 4\pi * 10^{-7}$ kg· s^{-2}· A^{-2}·m
$c_0 = 2.998 * 10^8$ m/s